NASA TT F-11,382

N68 13480

STUDY OF HEAT TRANSFER OF A TURBULENT FLUID FLOW IN A PLANE CURVED CHANNEL

D.A.Nusupbekova and B.P.Ustimenko

N68-13480

Translation of "Issledovaniye teploobmena pri turbulentnom techniі zhidkosti v ploskom krivolineynom kanale".
Vestnik Akademii Nauk Kazakhskoy SSR, Vol.22, No.1, pp.47-54, 1966.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 DECEMBER 1967

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM THE BEST COPY FURNISHED US BY THE SPONSORING AGENCY. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE.

STUDY OF HEAT TRANSFER OF A TURBULENT FLUID FLOW IN A PLANE CURVED CHANNEL

D.A.Nusupbekova and B.P.Ustimenko

ABSTRACT. Results of an experimental study of heat transfer in a turbulent flow of an incompressible fluid in a plane curved channel, at Reynolds numbers from 50,000 to 178,000. The temperature profile and the coefficient of heat transfer are determined, showing that heat transfer in a plane curved channel (R = 1.15) is roughly 20 to 25% greater than in comparable plane straight channels, and that the coefficient of heat transfer on the outside wall of a curved channel is roughly two times as high as that on its inside wall. Hydrodynamic formulas of heat transfer in a well developed turbulent flow in a plane curved channel are set up. The results obtained with the formulas agree well with experiments.

In various papers (Ref.1, 2, 3, 4) results of an experimental investigation of the hydrodynamics of turbulent flow of an incompressible fluid in a plane curved channel are presented.

A theoretical solution of the problem of resistance and heat transfer in the laminar flow of fluids in such channels, as well as a comparison with straight channels, was given earlier (Ref.5).

This paper presents the main results of an experimental investigation of heat transfer in turbulent flow of an incompressible fluid in a plane curved channel. The results are compared with heat transfer in straight ducts and with the results of an approximate calculation based on application of the hydrodynamic theory of heat transfer to the flow considered.

1. Experimental Setup and Procedure.

The heat transfer of a viscous incompressible fluid in turbulent flow in a plane curved channel was investigated in a setup described in detail elsewhere (Ref.4). The average radius of curvature of the channel was r_{av} = 215 mm. The walls of the channel were spaced 30 mm apart and were 300 mm high, with the ratio of outer r_2 to inner r_1 radii $\left(R = \frac{r_2}{r_1}\right)$ being 1.15. A 12-kw electric radiator was used to heat the flow ahead of the convergent duct. The temperature of the flow was controlled with a variable autotransformer connected in parallel in the circuit of the electric heater.

---

* Numbers in the margin indicate pagination in the foreign text.

To maintain a constant temperature of its external and internal walls, the channel was provided with two autonomous cooling jackets within which water was circulated. The wall temperature was controlled with nichrome-constantan thermocouples attached to the channel at the sites of the cross sections over which the temperature field was to be measured. The maximum variation in wall temperature over the test section of the channel did not exceed 0.5°C. The temperature profiles at the cross sections were measured with special probes consisting of two thermocouples each. These probes were shifted by means of microcoordinate spacers with an accuracy to within 0.01 mm. The thermocouples were made of 0.1 mm diameter nichrome and constantan wire with silk insulation and placed in a syringe needle of outside diameter 1.2 mm. The thermocouple junctions protruded 10 mm against the direction of flow and were spaced 2 mm apart, thus permitting measuring the temperature profile in the vicinity of the channel walls. The coordinate origin, as done also in measuring total head, was determined from the instant of loss of contact between the thermocouple and the channel wall. All thermocouples were connected across a switch to the circuit of a PPTN-1 potentiometer equipped with a M-25/7 mirror galvanometer as the null-point instrument. The thermocouples were calibrated in a thermostat with respect to a standard laboratory thermometer, correct to within 0.1°C. The errors involved in measuring the temperature profile and determining the Nusselt number were estimated at not more than 1 and 10%, respectively. /48

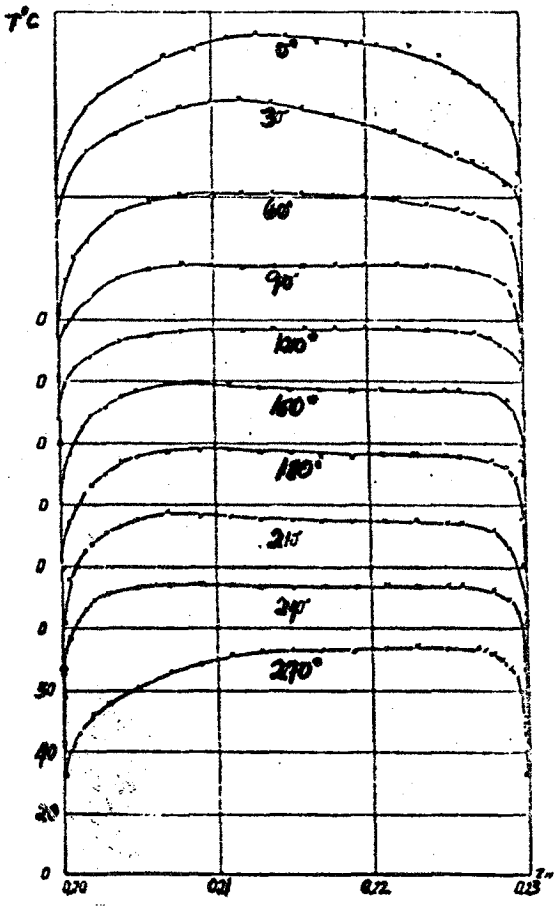

Fig.1 Deformation of Temperature Profile over the Length of the Channel, for Re = 77,600.

2. Experimental Results

All measurements were performed with the unit working in the steady-state regime, which usually set in within 3 - 4 hrs after start-up. The experiments included monitoring of the readings of the wall-temperature thermocouples and also of the thermocouples and Pitot tubes placed in the flow.

The Reynolds number, as calculated from the equivalent diameter, varied within the limits from 50,000 to 178,000. The temperature of the channel walls was maintained constant at 20 - 25°C in the various experiments. The temperature difference between the channel walls and the flow was selected within the range of 60 - 90°C.

Figure 1 shows the deformation of temperature distribution over the cross sections, as a function of increasing distance from the straight section of

the channel. The temperature profile changes from a symmetric profile, corresponding to the straight channel, to a profile (beginning roughly with $\varphi \approx 90^\circ$) corresponding to the stabilized fluid flow in a plane curved channel. Within that region ($\varphi > 90^\circ$), a fixed distribution of the relative excess temperature $\frac{t - t_w}{t_m - t_{cm}}$ sets in and subsequently remains unchanged at further increase in the angle φ (Fig.2a). The flow temperature is quasi-constant in the center of the flow and rapidly decreases on approach to the channel walls.

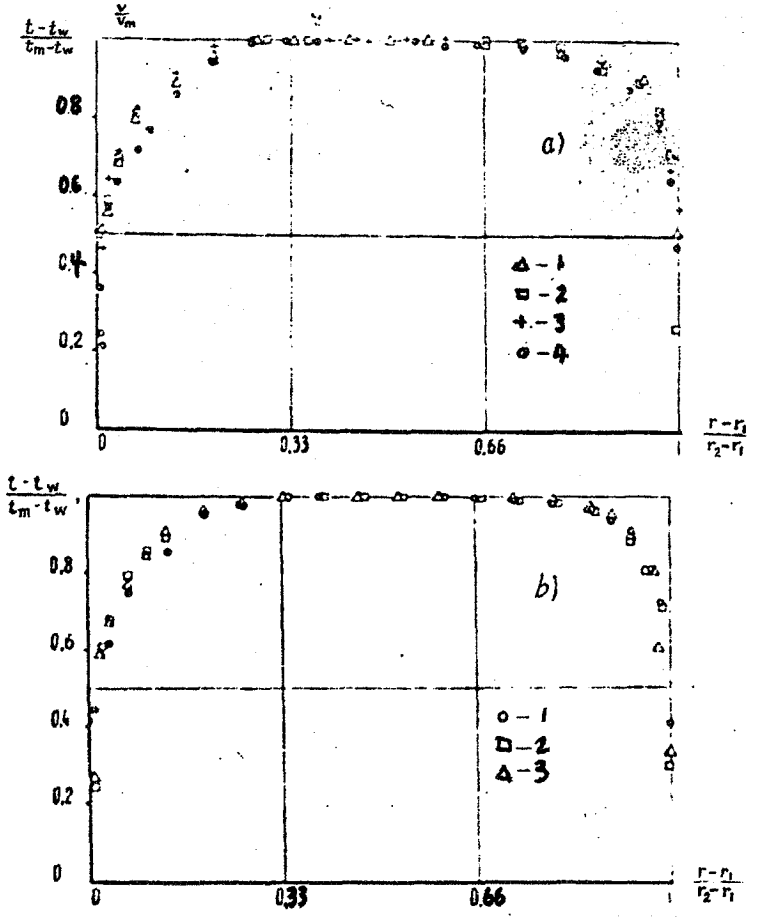

Fig.2 Distribution of Dimensionless Temperature Profile over the Channel Cross Section, for Various Angles (a) and Various Re Numbers (b)

a) 1 - $\varphi = 150^\circ$; 2 - $\varphi = 180^\circ$; 3 - $\varphi = 210^\circ$; 4 - $\frac{U}{U_m}$;

b) 1 - Re = 50,000; 2 - Re - 104,000; 3 - Re = 178,000 for $\varphi = 150^\circ$.

For comparison, Fig.2a presents the profile of relative velocity U/U_m corresponding to the same flow regime (Re = 77,600). As can be seen from this

diagram, the temperature and velocity profiles are close in shape and may be considered as roughly similar.

Profiles of relative temperature in the stabilized flow region are shown in Fig.2b for various values of the Re number. As implied by this plot, the effect of the Re number on the temperature distribution is virtually negligible, within the investigated range of variations (50,000 to 178,000).

Let us now consider the rate of heat transfer in turbulent flow in a plane curved channel. Let us determine the Nusselt number, characterizing the total release of heat from both channel walls to the flow, on the basis of the formula

$$Nu_0 = \frac{d_{equiv}}{\lambda} = \frac{q\,2(r_2-r_1)}{\lambda(\bar{t}-t_w)}, \qquad (1)$$

where $\alpha = \frac{q_0}{t - t_w}$ is the total coefficient of heat transfer in the channel; $d_{equiv} = 2(r_2 - r_1)$ is the equivalent diameter; λ is the coefficient of heat conduction; $q_0 = \frac{q_1 r_1 - q_2 r_2}{r_1 - r_2}$ is the unit heat flux q averaged over the channel surface; q_1, q_2 are the unit heat fluxes on the external and internal walls of the channel.

The mean excess temperature $\bar{t} - t_w$ was derived from the relation

/50

$$\bar{t} - t_w = \frac{\int_{r_1}^{r_2} u(t - t_w)\,dr}{\int_{r_1}^{r_2} u\,dr}. \qquad (2)$$

To estimate the rate of heat transfer at the internal and external walls of the channel, the flow can be conditionally divided into two regions: from the internal wall to the surface of maximum temperature with the radius r_m, and from r_m to the external wall. Such a division is possible, since the rate of heat flow across the surface of radius r_m is zero.

The Nusselt number, determined from the coefficient of heat transfer α_1 referred to the internal wall, is written in the form

$$Nu_1 = \frac{\alpha_1 d_{equiv}}{\lambda} = \frac{2(r_m - r_1)q_1}{\lambda(\bar{t}_1 - t_{cm})}. \qquad (3)$$

Then, the mean excess temperature is calculated from the formula

$$\bar{t}_1 - t_{cm} = \frac{\int_{r_1}^{r_m} u(t - t_{cm})dr}{\int_{r_1}^{r_m} u dr}, \tag{4}$$

while the density of heat flux q_1 is

$$q_1 = \int_{r_1}^{r_m} \rho C_p u \frac{\partial T}{r_1 d\bar{\tau}} dr. \tag{5}$$

The Nusselt number referred to the external wall of the channel is similarly derived:

$$Nu_2 = \frac{\alpha_2 l_{2equiv}}{\lambda} = \frac{2(r_2 - r_m)q_2}{\lambda(\bar{t}_2 - t_{cm})}, \tag{6}$$

where

$$\bar{t}_2 - t_{cm} = \frac{\int_{r_m}^{r_2} u(t - t_{cm})dr}{\int_{r_m}^{r_2} u dr} \tag{7}$$

and

$$q_2 = \int_{r_m}^{r_2} \rho C_p u \frac{\partial T}{r_2 d\bar{\tau}} dr. \tag{8}$$

Figure 3a shows the variation in the Nusselt numbers Nu_0, Nu_1, and Nu_2 /51 over the length of the channel with respect to a specific Reynolds number (Re = = 77,600). As indicated by the diagram, the thermal stabilization of the flow sets in at the channel segment corresponding to the angle $\varphi \geq 90^\circ$.

The dependence of the Nusselt number Nu_0 on the Reynolds number is shown in Fig.3b. The broken line (2) in this diagram is plotted from the formula

$$Nu = 0.023 Pr^{0.4} Re^{0.8}, \tag{9}$$

which is recommended for an annular straight duct (Ref.7).

The Nusselt number for heat transfer in a plane curved channel is higher (by approximately 20 - 25%) than for heat transfer in straight ducts and in plane

straight channels (Ref.8) and is in agreement with the increase in the coefficient of drag in the curved channel (Ref.4).

The Nusselt numbers Nu_1 and Nu_2 are plotted in Fig.3c as a function of the Reynolds number. As in the case of the distribution of tangential friction stress described elsewhere (Ref.4), the Nusselt number for the external channel wall is higher (by a factor of roughly 2) than for the internal channel wall. /52

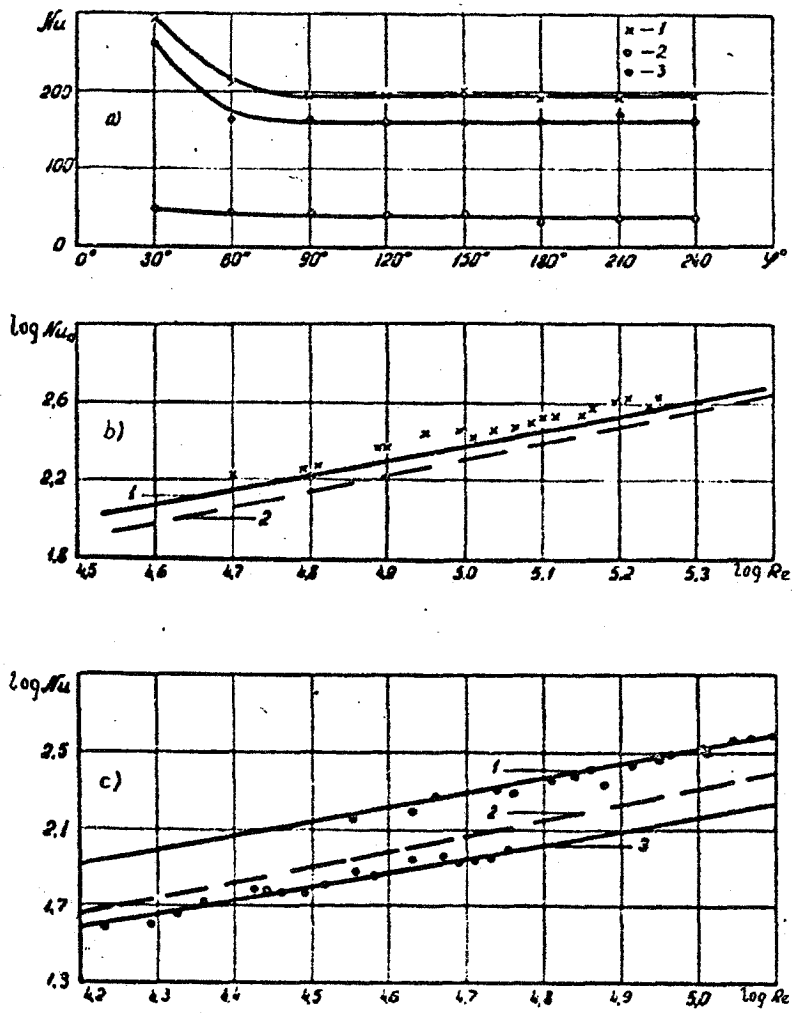

Fig.3 Distribution of Nusselt Numbers Nu_0, Nu_1, Nu_2 over the Channel Length (a) and for Various Reynolds Numbers (b). a) 1 - Nu_0; 2 - Nu_1; 3 - Nu_2; b) 1 and 2 - Calculated from formulas (11) and (9); c) 1, 2, and 3 - Calculated from formulas (14), (9), and (15).

It is of interest to note that a diametrically opposite ratio of the temperature intensity on the internal (convex) to the external (concave) walls has been derived for annular straight channels (Ref.6) as well as in calculations of

laminar heat transfer in plane curved channels (Ref.5). In such cases the coefficient of heat transfer for the internal wall is higher than for the external.

This qualitative difference is associated with the marked effect of the field of centrifugal forces on the turbulent pulsating motion in curvilinear flow (Ref.9).

The field of centrifugal forces intensifies the processes of turbulent transfer at the external wall of the plane curved channel (region of unsteady-state flow) while at the same time retarding them at the internal wall (region of steady-state flow) (Ref.4). The flow in a plane curved channel may exemplify the case where conclusions as to flow patterns, deduced from solution of the laminar problem, are qualitatively different from the patterns experimentally observed in turbulent flow.

3. Calculation of Heat Transfer

During the flow of fluids at Prandtl numbers close to unity in straight channels, the velocity and temperature are roughly similar if their boundary conditions are the same (constant wall temperature)

$$\frac{u}{u_m} \approx \frac{t-t_{cm}}{t_m-t_{cm}}. \tag{10}$$

As shown by experiments, an analogous similarity is observed in a plane curved channel with the boundary condition t_{cm} = const (Fig.2a). In this case, the rate of heat transfer can be determined without necessitating a direct integration of the equations of heat transfer.

Let us transform eq.(1) for the Nusselt number Nu_0 with the aid of the relations $q_1 = \lambda\left(\frac{\partial t}{\partial r}\right)_{r=r_1}$; $q_2 = -\lambda\left(\frac{\partial t}{\partial r}\right)_{r=r_2}$ in the form

where

$$\theta = \frac{t-t_{cm}}{t_m-t_{cm}}.$$

Making use of the similitude condition (10), we have

$$Nu_0 = \frac{2(r_2-r_1)}{r_2+r_1}\,\frac{r_1\left(\frac{\partial u}{\partial r}\right)_{r_1} - r_2\left(\frac{\partial u}{\partial r}\right)_{r_2}}{\bar{u}} = \frac{2(r_2-r_1)}{r_2+r_1}\cdot\left[\frac{r_1\frac{u_{*1}^2}{\nu}+r_2\frac{u_{*2}^2}{\nu}}{\bar{u}}\right] \tag{11}$$

where $\bar{u} = \frac{\int_{r_1}^{r_2} u\,dr}{r_2-r_1}$ is the mean (averaged over the cross section) velocity u_{*_1} =

$= \sqrt{\frac{\tau_{w_1}}{\rho}}$; $u_{*_2} = \sqrt{\frac{\tau_{w_2}}{\rho}}$; τ_{w_1}, τ_{w_2} are the tangential friction stresses on the internal and external channel walls, respectively. The Stanton number (St_0) then is determined in the following manner: /53

$$St_0 = \frac{Nu_0}{Pe_0} = \frac{r_1\tau_{w_1} + r_2\tau_{w_2}}{\rho\bar{u}^2(r_2 + r_1)}, \tag{12}$$

where

$$Pe_0 = \frac{\bar{u}2(r_2 - r_1)}{a}.$$

Making use of the condition of equilibrium between the forces of tangential friction stress and the forces of pressure

$$r_1\tau_{w_1} + r_2 \cdot \tau_{w_2} = \frac{\partial P}{\partial \varphi}(r_2 - r_1),$$

as well as of the formula for the coefficient of drag

$$\frac{\partial P}{\partial \varphi} = \xi_0 \frac{\rho\bar{u}^2}{8} \frac{r_1 + r_2}{(r_2 - r_1)},$$

we can rewrite eq.(14) in the form

$$St_0 = \frac{\xi_0}{8} \quad \text{or else} \quad Nu_0 = \frac{\xi_0}{8} Pe_0. \tag{13}$$

Thus, in the case of turbulent heat transfer in a plane curved channel, the Stanton number is related to the drag coefficient just as in straight channels.

The relations for the numbers Nu_1 and Nu_2 (St_1 and St_2) can be similarly derived from the correspondingly determined drag coefficients ξ_1 and ξ_2:

$$Nu_1 = \frac{\xi_1}{8}\left(1 + \frac{r_m}{r_1}\right)Pe_1, \quad St_1 = \frac{\xi_1}{8}\left(1 + \frac{r_m}{r_1}\right), \tag{14}$$

$$Nu_2 = \frac{\xi_2}{8}\left(1 - \frac{r_m}{r_2}\right)Pe_2 \quad St_2 = \frac{\xi_2}{8}\left(1 - \frac{r_m}{r_2}\right) \tag{15}$$

where

$$\xi_1 = \frac{\partial P}{\partial \varphi} \frac{8}{\rho\bar{u}^2_1} \frac{r_m - r_1}{r_m + r_1},$$

$$\xi_2 = \frac{\partial P}{\partial \varphi} \frac{8}{\rho\bar{u}^2_2} \frac{r_2 - r_m}{r_2 + r_m}.$$

For the investigated curved channel (R = 1.15), the drag coefficients ξ_0, ξ_1, and ξ_2 can be written in the form

$$\xi_0 = \frac{0.471}{Re^{0.25}}, \tag{16}$$

$$\xi_1 = \frac{0.130}{Re^{0.25}}, \tag{17}$$

$$\xi_2 = \frac{0\ 242}{Re^{0.25}}. \tag{18}$$

The selected characteristic dimension for the Reynolds number in eqs.(16), (17), and (18) was the doubled widths of the plane channel and of the regions /54 at the internal and external channel walls, respectively.

The solid line in Figs.3b and 3c represents the dependence of Nu_0, Nu_1, and Nu_2 on the Reynolds number, as calculated from eqs.(13), (14), and (15). As can be seen from this diagram, the calculation satisfactorily fits the experimental findings.

In conclusion, let us enumerate the principal results of our investigation.

Heat transfer during the turbulent flow of an incompressible fluid in a plane curved channel was experimentally investigated over the range of variations in Reynolds number from 50,000 to 178,000.

Detailed results on temperature profiles and heat transfer coefficients were obtained. The rate of heat transfer in a plane curved channel (R = 1.15) is approximately 20 - 25% higher than in plane and annular straight ducts.

It is shown that the coefficients of transfer of heat to the external wall are markedly higher (by about a factor of 2) than those to the internal wall. This has to do with the intensification of transfer processes at the external wall (region of nonsteady-state flow) and their retardation at the internal wall (region of steady-state flow) owing to the action of the centrifugal force field.

Calculations of heat transfer based on the application of the hydrodynamic theory of heat transfer to a fully developed turbulent flow in a plane curved channel are presented. The formulas derived are in good agreement with experimental findings.

REFERENCES

1. Wattendorf, F.L.: A Study of the Effect of Curvature on Fully Developed Turbulent Flow. Proc. Roy. Soc., Ser.A, Vol.148, 1935.
2. Braun, O.G. and Marris, A.V.: Turbulent Flow of Water in Plane Curved Channels of Finite Depth (Turbulentioye techeniye vody v ploskikh krivolineynykh kanalakh konechnoy glubiny). Teknicheskaya Mekhanika, Vol.3, 1963 (also Transactions of the American Society of Mechanical Engineers).
3. Marris, A.W.: Radial Distribution of Temporal - Mean Peripheral Velocity and Pressure for Fully Developed Turbulent Flow in Curved Channels.

J. Basic Eng. Trans. ASME, Ser.D, Vol.82, No.3, September, 1960
4. Nusupbekova, D.A. and Ustimenko, B.P.: Experimental Investigations of the Hydrodynamics of a Plane Curved Channel (Eksperimental'noye issledovaniye gidrodinamiki ploskogo krivolineynogo kanala). Probl. Teploenerg. Prikl. Teplofiz., Vol.2, Alma-Ata, 1965.
5. Nusupbekova, D.A. and Ustimenko, B.P.: Calculation of Laminar Convective Heat Transfer in a Plane Curved Channel (Raschet laminarnogo konvektivnogo teploobmena v ploskom krivolineynom kanale). Vestn. Ak. Nauk KazSSR, No.7, p.232, 1964.
6. Barlybayev, Kh.A.: Investigation of Heat Transfer and Hydrodynamics of Turbulent Flow in Annular Channels (Issledovaniye teploobmena i turbulentnogo potoka v kol'tsevykh kanalakh). Vestn. Ak. Nauk KazSSR, No.3, p.216, 1963.
7. McAdams, V.Kh.: Heat Transfer (Teploperedacha). Metallurgizdat, 1961.
8. Novothy, J.L., McComas, S.T., Sparrow, E.M., and Eckert, E.R.G.: Heat Transfer for Turbulent Flow in Rectangular Ducts with Two Heated and Two Unheated Walls. Am. Inst. Chem. Engs. Journal, Vol.10, No.4, 1964.
9. Zmeykov, V.N., Kel'manson, I.A., and Ustimenko, B.P.: Investigation of Turbulent Flow Structure in an Annular Channel with an Internal Rotating Cylinder (Issledovaniye turbulentnoy struktury potoka v kol'tsevom kanale s vnutrennim vrashchayushchimoya tsilindrom). In Coll.: Prikl. Teplofiz., No.1, Alma-Ata, 1964.

Translated for the National Aeronautics and Space Administration by the O.W.Leibiger Research Laboratories, Inc.